Enrollment No: ______________________ **Exam Seat No:**______________________

C. U. SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Discrete Mathematics

Subject Code: 4TE04DSM2 **Branch: B.Tech (CE)**

Semester: 4 **Date: 02/05/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) In a Lattice, $a \leq b$ and $b \leq c$ then
(a) $b \leq c$ (b) $c \leq a$ (c) $a \leq c$ (d) None of these

b) An Equivalent Realation is
(a) Reflexive (b) Symmetric (c) Transitive (d) all of these

c) Graph is collection of ______
(a)Equation (b)Row and columns (c)Vertices and edges (d) None of these

d) A tree is ___________.
(a) disconnected acylic graph (b) connected acylic graph
(c) may be connected or disconnected (d) None of these

e) If $G = \{1, -1, i, -i\}$ is a group under multiplication then $O(G)$ is _____
(a) 1 (b) 2 (c) 3 (d) 4

f) If m_i and m_j are distinct minterms in n −variable then______
(a) $m_i * m_j = 0$ (b) $m_i * m_j = 1$ (c) $m_i \oplus m_j = 0$ (d) $m_i \oplus m_j = 1$

g) A Complemented distributive lattice is called _____
(a) Boolean algebra (b) Modular lattice
(c) Bounded lattice (D) Complete lattice

h) The negation of $\forall x, P(x)$ is _____
(a) $\exists x, P(x)$ (b) $\exists x, \sim P(x)$ (c) $\forall x, \sim P(x)$ (d) $\nexists x, P(x)$

i) If n objects are placed in m places for $m < n$, then one of the places must contains at least _______ objects
(a) $\left[\frac{n-1}{m}\right] + 1$ (b) $\left[\frac{n+1}{m}\right] - 1$ (c) $\left[\frac{n-1}{m}\right] - 1$ (d) $\left[\frac{n+1}{m}\right] + 1$

j) True or false: Every cyclic group is abelian.

k) Define: Strongly Connected diagraph

l) State Pigeonhole principle.

m) Is $< S_{75}, GCD, LCM, 1, 75 >$ Complemented lattice?

n) True or false: I and O are only complement of each other.

Attempt any four questions from Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	a)	Show that $\langle R, min, max \rangle$ is a lattice	**(06)**
	b)	Prove that any two cosets of a subgroup are either disjoint or identical.	**(04)**
	c)	For a lattice $< S_{1001}, D >$, Find cover of each element and Also Draw the Hasse diagram.	**(04)**
Q-3		**Attempt all questions**	**(14)**
	a)	Find Join-irreducible, atom, meet-irreducible and anti-atom for $\langle S_{70}, D \rangle$.	**(05)**
	b)	Let $< L,*,\oplus,0,I >$ be a lattice and $a,b,c \in L$ then following conditions are equivalent *i)* $a*(b \oplus c) = (a*b) \oplus (a*c)$ *ii)* $a \oplus (b*c) = (a \oplus b)*(a \oplus c)$	**(05)**
	c)	Show that $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.	**(04)**
Q-4		**Attempt all questions**	**(14)**
	a)	Find all node base of the following diagraph shown in the figure. Also give paths from vertex H to D.	**(06)**

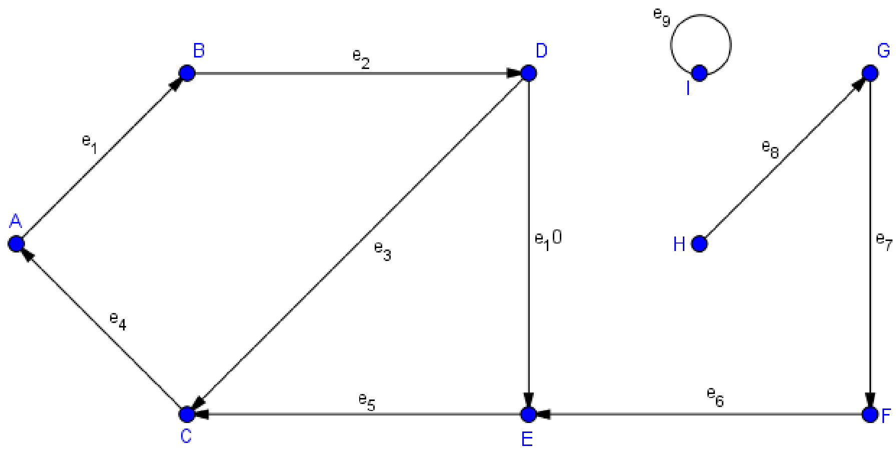

Figure- 1

	b)	Prove that the number of vertices of odd degree in a undirected graph is always even.	**(05)**
	c)	State Handshaking theorem. Also draw an undirected graph with 4 vertices such that all the vertices have three degree	**(03)**
Q-5		**Attempt all questions**	**(14)**
	a)	State and prove stone's representation theorem.	**(07)**
	b)	State and prove Lagrange's theorem on group.	**(07)**
Q-6		**Attempt all questions**	**(14)**
	a)	Find all minterms of a Boolean algebra with free variables x_1, x_2, x_3.	**(05)**
	b)	Prove that $1+2+3+\cdots+n = \frac{n(n+1)}{2}$ by using Mathematical induction. where n be a positive integer.	**(04)**
	c)	From the following adjacency matrix, find the out degree and in degree of each node. Also verify your answer by drawing digraph and its	**(05)**

adjacency matrix

$$\begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 2 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

Q-7 **Attempt all questions** **(14)**

a) Show that the set of cube roots of unity form a group under multiplication. **(05)**

b) Show that $\sim r$ is a valid conclusion from the premises $p \Rightarrow\sim q, r \Rightarrow p, q$ with truth table and without truth table. **(05)**

c) Write the Boolean xpressions $x_1 * x_2$ in an equivalent sum of products canonical form in three variables x_1, x_2, x_3. **(04)**

Q-8 **Attempt all questions** **(14)**

a) Show that $< S_{30}, *, \oplus >$ and $< P(A), \cap, \cup >$ are isomorphic lattices for $A = \{a, b, c\}$ **(07)**

b) Let $\langle L, *, \oplus, ', 0, 1 \rangle$ be a complemented lattice and for any $a, b \in L$ then prove that $a \le b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \le a' \Leftrightarrow a' \oplus b = 1$ **(07)**